

Using the Poisson Distribution

The following problem will be used to practice using the Poisson distribution in the calculator:

A smoothie shop sells on average 80 strawberry, 47 raspberry, and 23 banana smoothies on any given day.

Problems:

- a.) What is the probability that the shop sells exactly 130 smoothies in a day?

- b.) What is the probability that the shop sells at most 85 strawberry smoothies in a day?

- c.) What is the probability that the shop sells more than 40 banana smoothies in a day?

*First, think : should the probabilities for each be relatively high, low, etc.? Does your answer make sense in the context of the problem?

The questions being asked indicate that this is a Poisson distribution because the experiment consists of counting the number of times an event will occur in a given interval. We know the probability of the event occurring is the same for each interval, and the number of occurrences in one interval is independent of the number of occurrences in other intervals. The interval here is one day and the formula for the probability is

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

where μ is the mean number of occurrences per interval unit and e is the irrational number equal to 2.718.

a.) P(130)

Here we want the probability of exactly selling 130 in any day. We can use the formula above, but why do that if we can use the calculator to do this probability for us using the *poissonpdf* function in the calculator. First, we know the mean number of smoothies sold in a given day is 150 (80+47+23). Now, in the calculator we input:

2nd > DISTR(on top of VARS) > scroll down to C: poissonpdf(>
ENTER > 150 , 130)

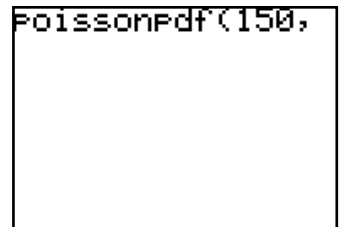
(Screen 1)

ENTER

(Screen 2)

0.008 (or about 0.865%) is the probability that the shop sells exactly 130 smoothies in a day.

Screen 1



Screen 2



b.) P(x ≤ 85)

Think: On average the shop sells 80 strawberry smoothies per day, so selling at most 85 is not unusual. Therefore, the probability should be relatively high.

The mean number of strawberry smoothies sold in a day is 80. The probability can be found by using the *poissoncdf* function.

Follow this sequence:

2nd > DISTR(on top of VARS) > scroll down to D:poissoncdf(>
ENTER > 80 , 85)

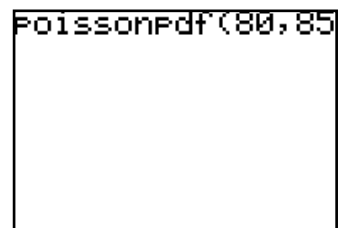
(Screen 3)

ENTER

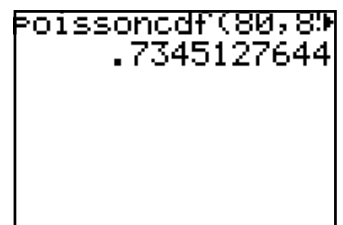
(Screen 4)

0.735 (or about 73.5%) is the probability that the shop sells at most 85 strawberry smoothies in a day.

Screen 3



Screen 4



c.) $P(x > 40)$

Think: On average the shop sells only 23 banana smoothies per day, so selling almost twice that amount is unusual. Therefore, the probability should be low. Again, think about what the question is asking. Here we will use the *poissoncdf* function in the calculator, by subtracting the probability of $P(x \leq 40)$ from one.

$$P(x > 40) = 1 - P(x \leq 40)$$

Follow this sequence:

1 -> 2nd **DISTR** > scroll down to **D: poissoncdf**(> **ENTER** > **23** , **40**)

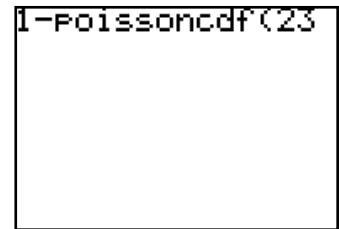
(Screen 5)

ENTER

(Screen 6)

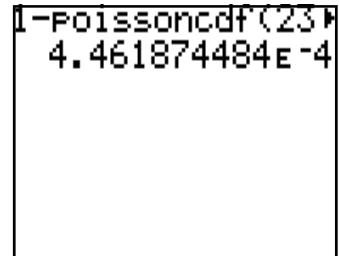
0.00045 (or about 0.0446%) is the probability of selling 40 banana smoothies in a day. Like we predicted, it is a low value.

Screen 5



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1-Poissoncdf(23
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Screen 6



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1-Poissoncdf(23)  
4.461874484E-4
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NOTE: Anytime when using the calculator, how do you know whether to use the *pdf* function or the *cdf* function?

Use the *cdf* function when you are looking for a sum over a range of values from 0 up to and including a specific number (less than or equal to).

Ex. $P(x \leq 10)$

Use the *pdf* function when you are looking for a single value of x (equality).

Ex. $P(x = 100) = P(100)$