Illowsky – Chapt. 9
Larson – Chapt. 7

Math 123, Fall 16, Midterm 4, Take-Home Name: Solutions
Instructor: Saba Gerami **Total: 100 Points**

Directions:
- Show all your work. You only receive half of the points if you do not explain your reasoning.
- You can use a graphing calculator.
- **Round to two decimal places.**

1. a) State two similarities between the normal curve and t-curve. *(4 points)*

Both are symmetrical
Both mean = median = mode
Both area under the curve is 1.

b) State two differences between the normal curve and χ^2 table.

normal is symmetrical, χ^2 is not.
normal has ⊕ & ⊖ values, χ^2 has only ⊕ values

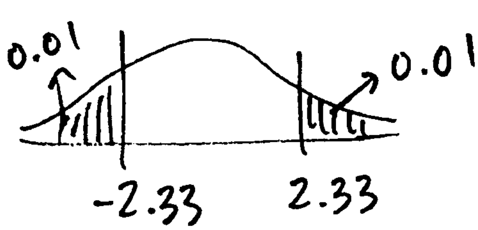
2. When does type I error and type II error happen? *(4 points)*

Type I: H_0 is rejected but H_0 is true
Type II: H_0 is not rejected but H_0 is false.

3. What is expected to happen to type I error if we increase level of significance (α)? *(2 points)*

α is prob of type I error
so if we inc α, Prob of type I error increases.

4. Find the critical value(s) and rejected region(s) for the two-tailed test of z-test with level of significance $\alpha = 0.02$. $\rightarrow$ each tail $= \frac{0.02}{2} = 0.01$ *(2 points)*



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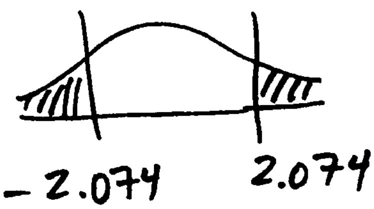
5. Find the critical value(s) and rejected region(s) for the two-tailed test of t-test with level of significance $\alpha = 0.05$ and $n = 23$. $\rightarrow df = 23-1 = 22$ *(2 points)*

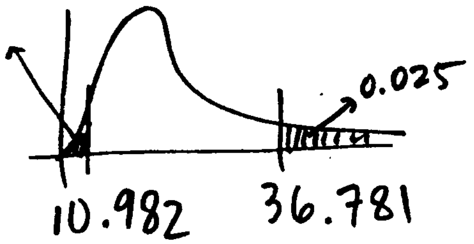
$\alpha = 0.05 \rightarrow$ one tail $= \frac{0.05}{2} = 0.025$



6. Find the critical value(s) and rejected region(s) for the two-tailed test of χ^2-test with level of significance $\alpha = 0.05$ and $n = 23$. $\rightarrow df = 22$ *(4 points)*

$0.025 \leftrightarrow 1 - 0.025 = 0.975$

$\alpha = 0.05 \rightarrow$ each tail $= \frac{0.05}{2} = 0.025$



$\chi^2_R = 36.781$

$\chi^2_L = 10.982$

In all of the following questions, number your steps as it is in your formula sheet. $\mu = 3.5$

7. A study says the mean time to recoup the cost of bariatric surgery is 3.5 years. You randomly select 30 surgery patients and find that the mean time to recoup the cost of their surgeries is 3.8 years. Assume the population standard deviation is 0.3 year. Is there enough evidence to doubt the study's claim at $\alpha = 0.01$? *(12 points)*

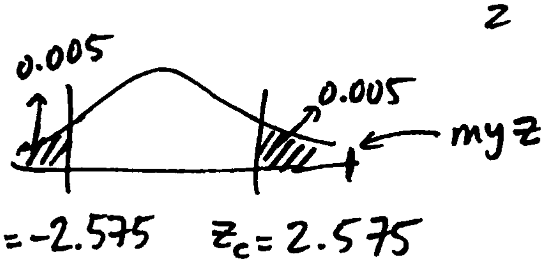
$n = 30$
$\bar{X} = 3.8$
$\sigma = 0.3$

Hypothesis Testing a:

1. Verify: Sample random ✓
   $n \geq 30$ ✓

2. $\begin{cases} H_0: \mu = 3.5 & \text{claim} \\ H_a: \mu \neq 3.5 & \rightarrow \text{two tailed} \end{cases}$

3. $\alpha = 0.01 \rightarrow$ one tail $= \frac{0.01}{2} = 0.005$

4. 

5. My $z = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \dfrac{3.8 - 3.5}{\frac{0.3}{\sqrt{30}}}$

   $= 5.48$ in rejected region

6. Reject H_0

7. Reject the claim.

$\sigma^2 \leq 30$

8. A fitness magazine advertises that the variance of monthly cost of a yoga session is not more than \$30 in Los Angeles. You work for a consumer advocacy group and are asked to test this claim. You find that a random sample of 16 yoga sessions has a variance cost of \$42.35. At $\alpha = 0.025$, do you have enough evidence to reject the magazine's claim if the population is normally distributed? *(12 points)*

$n=16$

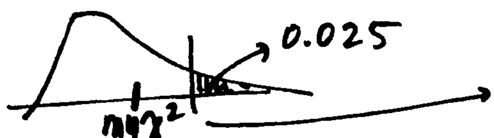
$s^2=42.35$

Hypothesis Testing α:

1- Verify: random sample ✓
pop. normal ✓

2- $\begin{cases} H_0: \sigma^2 \leq 30 & \text{claim} \\ H_a: \sigma^2 > 30 & \text{right-tailed} \end{cases}$

3- $\alpha = 0.025$, $df = 16-1=15$

4- 
$\chi^2 = 27.488$

5- My $\chi^2 = \dfrac{(n-1)(s^2)}{\sigma^2} = \dfrac{15(42.35)}{30}$
$= 21.175$ not in rejected region

6- Fail to reject H_0

7- Cannot reject the claim.

9. A coffee shop owner claims that 80% of coffee drinkers think that the taste of a shop's coffee is the most important factor where they purchase their coffee. In a random sample of 36 coffee drinkers, 28 people think that the taste of a shop's coffee is the most important factor where they purchase their coffee. At $\alpha = 0.10$, is there enough evidence to support the owner's claim? *(12 points)*

$P=0.8$
$n=36$
$\hat{P} = \frac{28}{36} = .78$
$X=28$

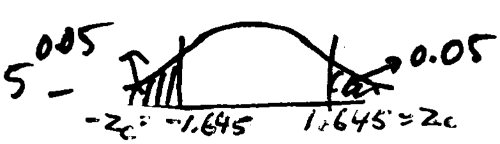
Hypothesis Testing $\underline{c}$:

1- $n=36$, $p=0.8$, $q=0.2$, $\hat{p} = \frac{28}{36} = 0.78$, $\hat{q} = 1-0.78=0.22$

2- $np = 36(0.8) = 28.8 \geq 5$ ✓ $nq = 36(0.2) = 7.2 \geq 5$ ✓

3- $\begin{cases} H_0: P=0.8 & \text{claim} \\ H_a: P \neq 0.8 & \text{two-tailed} \end{cases}$

4- $\alpha = 0.10 \rightarrow$ one tail $= \frac{0.1}{2} = 0.05$

5- 

6- My $z = \dfrac{\hat{p}-p}{\sqrt{pq/n}} = \dfrac{0.78-0.8}{\sqrt{0.8 \times 0.2/36}} = -0.2$

not in rejected region

7- Fail to reject H_0

8- Cannot reject the claim

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$\mu \leq 2020$

10. A researcher claims that the mean rent of a one-bedroom apartment in Santa Maria is at most \$2,020. In a random sample of 30 one-bedroom apartment in Santa Maria, the mean is \$2,050 and the standard deviation is \$152. At $\alpha = 0.05$, is there enough evidence to reject the claim? *(12 points)*

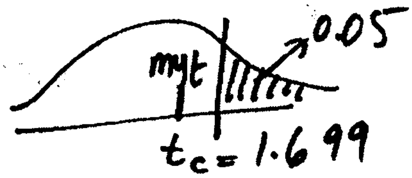
$n = 30$
$\bar{x} = 2050$
$s = 152$

Hyp. Testing b :

1. Verify: sample random ✓
   $n \geq 30$ ✓

2. $\begin{cases} H_0: \mu \leq 2020 & \text{claim} \\ H_a: \mu > 2020 & \text{right-hand} \end{cases}$

3. $\alpha = 0.05$
   $df = 29$

4. 

5. My $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{2050 - 2020}{152/\sqrt{30}}$
   $= 1.08$ not in rejected region

6. fail to reject H_0

7. Cannot reject the claim

11. A French restaurant claims that the standard deviation of the lengths of serving time is 3 minutes. A random sample of 60 serving times has a standard deviation of 4.1 minutes. At $\alpha = 0.01$, is there enough evidence to reject the restaurant's claim? Assume the population is normally distributed. *(12 points)*

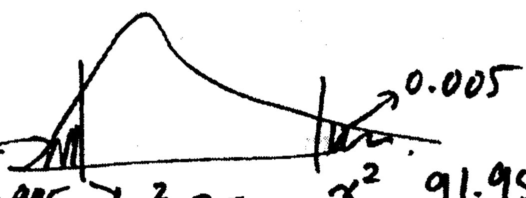
$\sigma = 3$
$n = 60$
$s = 4.1$

Hyp. Testing d:

1. Verify: Sample random ✓
   Pop normal ✓

2. $\begin{cases} H_0: \sigma = 3 & \text{Claim} \\ H_a: \sigma \neq 3 & \text{two-tail} \end{cases}$

3. $\alpha = 0.01 \rightarrow$ each-tail $= \dfrac{0.01}{2} = 0.005$
   $df = 60 - 1 = 59 \rightarrow$ use 60 (closest)

4. 

5. my $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2} = \dfrac{59(4.1)^2}{3^2}$
   $= 110.20$ in rejected region

6- Reject H_0

7- Reject the claim

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$\rightarrow P < 0.25, q = 0.75$

12. A medical researcher says that less than 25% of U.S. adults eat organic food. In a random sample of 500 U.S. adults, 19.3 % say that they eat organic. At $\alpha = 0.05$, is there enough evidence to support the researcher's claim? $n = 500$

(12 points) $\hat{p} = 0.193$

Hypothesis Testing C:

1. $n = 500$, $p = 0.25$, $q = 0.75$, $\hat{p} = 0.193$, $\hat{q} = 1 - 0.193 = 0.807$

2. $np = 500(0.25) = 125 \geq 5 \checkmark$, $nq = 500(0.75) = 375 \geq 5 \checkmark$

3. $\begin{cases} H_0: P \geq 0.25 \\ H_a: P < 0.25 \end{cases}$ claim, left handed

4. $\alpha = 0.05$

5. 0.05

$-z_c = -1.645$

6. my $z = \dfrac{\hat{p} - p}{\sqrt{pq/n}} = \dfrac{0.193 - 0.25}{\sqrt{\dfrac{0.25 \times 0.75}{500}}} = -2.94$ in rejected region

7. Reject H_0

8. We support the claim.

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