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A

Illowsky – Chapt. 9 & 10

Larson – Chapt. 7 & 8

Math 123 Exam 4

SHOW ALL WORK

Name

1. You do a left-tailed test and get standardized test statistic  $Z = -0.94$ . Find the P-value.

$$P\text{-value} = \text{normal cdf}(-10000, -0.94) \\ \approx 0.1736$$

2. The P-value for a hypothesis test is  $P = 0.0856$ . Would you reject  $H_0$  at a 10% level of significance? Explain very clearly.

$$\alpha = 0.10$$

$$0.0856 < 0.10$$

$$P\text{-value} \leq \alpha$$

Yes; reject  $H_0$

3. If you conduct 200 hypothesis tests at  $\alpha = 1\%$ , how many times would you expect to make a Type I Error (i.e. reject  $H_0$  when  $H_0$  is true)?

$\alpha =$  maximum acceptable risk of a Type I error

$$\alpha = 0.01$$

$$200 \times 0.01 = \boxed{2}$$

4. You are testing the claim that the mean age of AHC students is 22.3 years old. Write a sentence that *very clearly* explains what a Type I Error would be, and a second sentence doing the same for a Type II Error.

Type I: A type I error occurs if the actual mean age of AHC students is 23.3 years old and you reject  $H_0$ .

Type II: A type II error occurs if the actual mean age of AHC students is not 22.3 years old and you fail to reject  $H_0$ .

For all hypothesis tests, for full credit you must show all 4 steps as outlined in class:

- Hypotheses/claim
  - Critical Value with picture showing Rejection Region
  - How you computed the STS
  - Conclusion re: both  $H_0$  and the claim
5. Use  $\alpha = 5\%$  to test the claim that the population mean age of AHC male students exceeds the population mean age of AHC female students. Assume that a random sample of 35 AHC males yielded a mean age of 24.3 with standard deviation 3.1, as compared to a random sample of 31 AHC females which yielded a mean age of 23.1 years with standard deviation 5.2.

$$\bar{x}_1 = 24.3$$

$$n_1 = 35$$

$$s_1 = 3.1$$

$$\bar{x}_2 = 23.1$$

$$n_2 = 31$$

$$s_2 = 5.2$$

$$\alpha = 0.05$$

$$d.f. = 30$$

$$a) H_0: \mu_1 \leq \mu_2$$

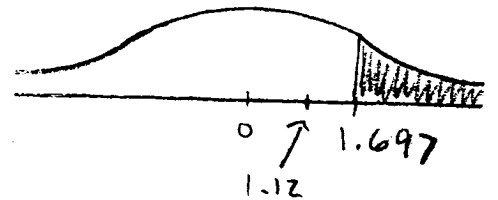
$$H_a: \mu_1 > \mu_2 \text{ (claim)}$$

\* Assume population variances not equal \*

$$b) t_0 = 1.697 \text{ (from table)}$$

$$c) \text{ STS: } t = \frac{(\bar{x}_1 - \bar{x}_2) - (0)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{24.3 - 23.1}{\sqrt{\frac{3.1^2}{35} + \frac{5.2^2}{31}}} = \boxed{1.12}$$



d) Do not reject  $H_0$

e) Do not support claim

6. A soda company claims that 17% of the population drinks their soda. To test this claim, you take a random sample of 105 people and find that 8 of them drink the company's soda. Test the company's claim at  $\alpha = 10\%$ .  $\alpha = 0.10$

Proportion

Valid? yes

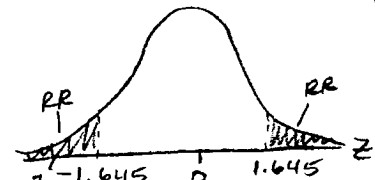
$$np_0 \geq 5$$

$$nq_0 \geq 5$$

$$a) H_0: p_0 = 0.17 \text{ (claim)} \quad H_a: p_0 \neq 0.17$$

$$b) z_0 = \pm 1.645 \text{ (from table)}$$

$$c) \text{ STS: } z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 \cdot q_0}{n}}} = \frac{0.076 - 0.17}{\sqrt{\frac{(0.17)(0.83)}{105}}} = \frac{-0.094}{.036658} = \boxed{-2.56}$$



d) Reject  $H_0$

e) Reject the claim

$$P\text{-value} = \overset{2*}{\text{normalcdf}}(-10000, -2.56)$$

$$= 0.0105$$

$$0.0105 < 0.10$$

$$P\text{-value} \leq \alpha \checkmark$$

claim:

$$p_0 = 0.17$$

$$n = 105$$

$$\hat{p} = \frac{8}{105}$$

$$\hat{p} = 0.076$$

$$q_0 = 0.83$$

2-tailed

7. A bank claims that the population standard deviation for the wait time to see a live teller is less than 40 seconds. A random sample of 25 customers yields a sample standard deviation of 28 seconds. Test the bank's claim at alpha = 5%.

Claim:  
 $\sigma_0 < 40$   
 $n = 25$   
 $s = 28$   
 $df = n - 1 = 24$

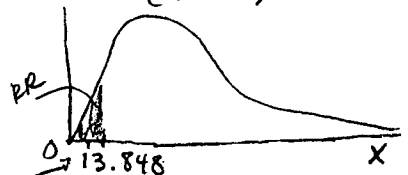
a)  $H_0: \sigma \geq 40 \text{ sec.}$   $H_a: \sigma < 40 \text{ sec. (claim)}$

b)  $\chi^2_0 = 13.848$  (from table) (chi square table)

c) STS:  $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(25-1)28^2}{40^2} = \frac{18,816}{1600}$

$= 11.76$

$\alpha = 0.05$   
 left-tailed  
 $(1-\alpha) = 0.95$



d) Reject  $H_0$   
 e) support the claim

$P\text{-value} = \chi^2_{cdf}(0, 11.76, 24)$   
 $= 0.0175$   
 $0.0175 < 0.05$   
 $P\text{-value} \leq \alpha \checkmark$

8. A counselor claims that the proportion of ACME College students that transfer to a 4 year university is less than the proportion of Ace College students that transfer to a 4 year university. A random sample of 200 ACME students shows that 27% transfer, as compared to 39% of a random sample of 200 Ace students. Test the counselor's claim using a 1% level of significance.

Claim:  
 $p_1 < p_2$   
 $n_1 = 200$   
 $n_2 = 200$   
 $\hat{p}_1 = 0.27$   
 $\hat{p}_2 = 0.39$   
 $x_1 = \hat{p}_1 n_1 = (0.27)(200) = 54$   
 $x_2 = \hat{p}_2 n_2 = (0.39)(200) = 78$   
 $\bar{p} = \frac{54 + 78}{200 + 200} = \frac{132}{400} = 0.33$   
 $\bar{q} = 1 - 0.33 = 0.67$

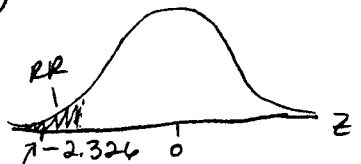
$p_1$  - ACME  
 $p_2$  - Ace

a)  $H_0: p_1 \geq p_2$   $H_a: p_1 < p_2$  (claim)

b)  $z_0 = -2.326$  \* invnorm(0.01)

c) STS:  $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\bar{p} \cdot \bar{q} (\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.27 - 0.39}{\sqrt{(0.33)(0.67)(\frac{1}{200} + \frac{1}{200})}}$   
 $= \frac{-0.12}{0.04702127}$   
 $= -2.55$

left-tailed



d) Reject  $H_0$   
 e) support the claim

$P\text{-value} = \text{normalcdf}(-10000, -2.55)$   
 $= 0.0054$   
 $0.0054 < 0.01$   
 $P\text{-value} \leq \alpha \checkmark$

Claim:  
 $\mu \leq 12.3 \text{ in.}$   
 $n = 17$   
 $\bar{x} = 11.9$   
 $\sigma = 2.7$  or  
 $s = 2.7$   
 $d.f. = 17 - 1 = 16$

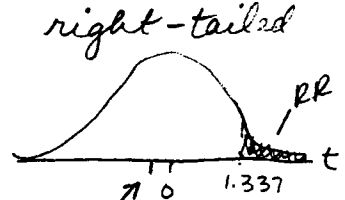
9. A biologist claims that the mean length of trout in a stream is at most 12.3 inches. A random sample of 17 trout from the stream yields a mean length of 11.9 inches with a standard deviation of 2.7 inches. Assuming trout lengths are normally distributed, test the claim using alpha = 10%.  $\alpha = 0.10$

$n < 30$   
 t-test

a)  $H_0: \mu \leq 12.3 \text{ in. (claim)}$   $H_a: \mu > 12.3 \text{ in.}$

b)  $t_0 = 1.337$  (table)

c) STS:  $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{11.9 - 12.3}{\frac{2.7}{\sqrt{17}}} = \frac{-0.4}{0.6548} = -0.6109$



d) Do not reject  $H_0$

e) Do not reject claim

$P\text{-value} = \text{tdf}(-0.6109, 10000, 16) = 0.7251$   
 $0.7251 > 0.10$   
 $P\text{-value} > \alpha \checkmark$

Claim:  
 $X_2 < X_1$   
 $0 < X_1 - X_2$

10. A researcher claims that taking a certain herb can reduce a person's blood pressure. Six subjects have their systolic blood pressure measured before and after taking the herb, with the results given below. Test the researcher's claim at alpha = 1%. You may assume blood pressure values are normally distributed.  $\alpha < 0.01$

right-tailed

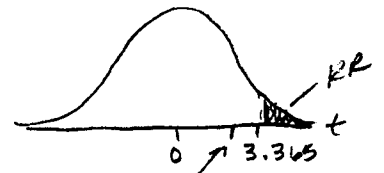
$\mu_d > 0$   
 $n = 6$   
 $s_d = 8.246$   
 (from calc.)  
 $d.f. = n - 1 = 5$   
 $\bar{d} = 10$

Systolic blood pressure (before herb):	145	132	117	150	129	122
Systolic blood pressure (after herb):	131	119	114	129	131	111

a)  $H_0: \mu_d \leq 0$   $H_a: \mu_d > 0$  (claim)

b)  $t_0 = 3.365$  (table)

c) STS:  $t = \frac{\bar{d} - 0}{\frac{s_d}{\sqrt{n}}} = \frac{10 - 0}{\frac{8.246}{\sqrt{6}}} = \frac{10}{3.36642} = 2.971$



d) Do not reject  $H_0$

e) Do not support the claim

$P\text{-value} = \text{tdf}(2.971, 10000, 5) = 0.0156$   
 $0.0156 > 0.01$   
 $P\text{-value} > \alpha \checkmark$