Spring 2014

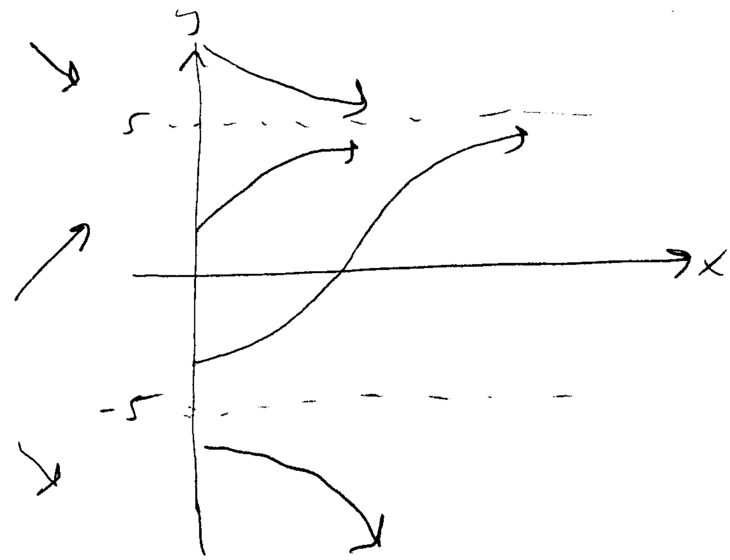
Math 184 Exam 2 SHOW ALL WORK Name Key

1. Find any equilibrium solutions, use calculus to find the y-intervals for which the solutions are increasing/decreasing and concave up/down, then sketch a phase portrait.

$y' = 25 - y^2$

$y' = (5-y)(5+y)$

Equil. solution at $y = \pm 5$



int.	y'	y
$(-\infty,-5)$	$-$	decr
$(-5,5)$	$+$	incr
$(5,\infty)$	$-$	decr

$y'' = -2y \cdot y'$

IP at $y=0$

if $y>5$, y'' is $+$

if $y<-5$, y'' is $-$

if $-5<y<0$, y'' is $+$

if $0<y<5$, y'' is $-$

2. Solve using separation of variables: $\frac{dy}{dx} = \sin(x)y^2 + e^x y^2$

$$\frac{dy}{dx} = y^2(\sin x + e^x)$$

$$\int y^{-2}\,dy = \int (\sin x + e^x)\,dx$$

$$-y^{-1} = -\cos x + e^x + c$$

$$\boxed{y = -\frac{1}{e^x - \cos x + c}} \quad \text{or} \quad \boxed{y = \frac{1}{\cos x - e^x + c}}$$

Scores

9	5 5	All turned in each time
8	3 4 5 0 7	
7	5 3 5 0 1 6	
6	5 6 3 6 3	
5	7 1 2 2 8	All missed some thing
4	8 6	
3		
2	4	

3. Solve using an integrating factor: $y' + x^4 y = x^4$

$① \; u = e^{\int x^4 dx} = e^{\frac{1}{5}x^5}$

$② \; \frac{d}{dx}\left[e^{\frac{1}{5}x^5} y\right] = e^{\frac{1}{5}x^5} \cdot x^4$

$w = \frac{1}{5}x^5$

$dw = x^4 dx$

$e^{\frac{1}{5}x^5} y = \int e^{\frac{1}{5}x^5} \cdot x^4 \, dx$

$= \int e^w dw$

$= e^{\frac{1}{5}x^5} + c \implies \boxed{y = 1 + Ce^{-\frac{1}{5}x^5}}$

4. Solve: $\frac{dy}{dx} = 3y + y^4$

Bernoulli! $v = y^{1-4} = y^{-3} \Rightarrow y = v^{-\frac{1}{3}}$

$-\frac{1}{3} v^{-4/3} \cdot \frac{dv}{dx} - 3v^{-1/3} = v^{-4/3}$

$\frac{dy}{dx} = -\frac{1}{3} v^{-4/3} \cdot \frac{dv}{dx}$

$\frac{dv}{dx} + 9v = -3$

$① \; u = e^{\int 9 dx} = e^{9x}$

$② \; \int \frac{d}{dx}\left[e^{9x} \cdot v\right] = \int -3e^{9x}$

$e^{9x} v = -\frac{1}{3}e^{9x} + c$

$v = -\frac{1}{3} + Ce^{-9x}$

$\boxed{y = \frac{1}{\sqrt[3]{-\frac{1}{3} + Ce^{-9x}}}}$

5. Solve: $(4xe^y - \sin(x))dx + (2x^2e^y - \frac{1}{y})dy = 0$

Exact $M_y = 4xe^y$

$N_x = 4xe^y$

$F_x = M: \; F = \int (4xe^y - \sin x)dx = 2x^2e^y + \cos x + h(y)$

$F_y = N: \; 2x^2e^y + h'(y) = 2x^2e^y - \frac{1}{y}$

$h(y) = \int -\frac{1}{y} dy = -\ln|y|$

$F(x,y) = 2x^2e^y + \cos x - \ln|y|$

so: $\boxed{2x^2e^y + \cos x - \ln|y| = c}$ is the solution

6. An amoeba population starts with 1000 amoeba and grows at a continuous rate of 6% per year. 500 amoeba per year are removed from the population at a continuous rate. Write an IVP (i.e. include initial conditions with your DE) that models the population at time t. DO NOT SOLVE.

$$\frac{dA}{dt} = 0.06A - 500, \quad A(0) = 1000$$

7. Solve: $y'' = (y')^2$

$v = y', \quad v' = y''$

$v' = v^2$

$\int v^{-2}\,dv = \int dx$

$-v^{-1} = x + C$

$v = -\frac{1}{x+C}$

$v = y' = \frac{dy}{dx} = -\frac{1}{x+C}$

$\int dy = \int -\frac{1}{x+C}\,dx$

$\boxed{y = -\ln|x+C_1| + C_2}$

8. Solve: $y''' - 3y'' - 4y' - 30y = 0$

$\lambda^3 - 3\lambda^2 - 4\lambda - 30 = 0$

$\lambda = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$\boxed{y = C_1 e^{5x} + e^{-x}\left[C_2 \cos\sqrt{5}x + C_3 \sin\sqrt{5}x\right]}$

5	1	-3	-4	-30
	↓	5	10	30
	1	2	6	0

$(\lambda - 5)(\lambda^2 + 2\lambda + 6) = 0$

$\lambda = 5, \quad \lambda = \frac{-2 \pm \sqrt{4 - 24}}{2} = \frac{-2 \pm 2\sqrt{5}\,i}{2}$

$= -1 \pm \sqrt{5}\,i$

E

9. Solve: $y'' - 10y' + 25y = 0$

$$\lambda^2 - 10\lambda + 25 = 0$$

$$(\lambda - 5)^2 = 0$$

$$\lambda = 5 \text{ (mult. 2)}$$

$$\boxed{y = c_1 e^{5x} + c_2 x e^{5x}}$$

10. Solve: $y'' - 4y = x^2$.

$$\lambda^2 - 4 = 0 \Rightarrow \lambda = \pm 2$$

$$y_h = c_1 e^{2x} + c_2 e^{-2x}$$

$$y_p = ax^2 + bx + c$$

$$y_p' = 2ax + b$$

$$y_p'' = 2a$$

$$2a - 4(ax^2 + bx + c) = x^2$$

$$-4ax^2 - 4bx + (2a - 4c) = x^2 + 0$$

$-4a = 1$ $\quad -4b = 0$ $\quad 2a - 4c = 0$

$a = -\frac{1}{4}$ $\quad b = 0$ $\quad -\frac{1}{2} - 4c = 0$

$c = -\frac{1}{8}$

$$\boxed{y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{4}x^2 - \frac{1}{8}}$$

11. Find a basis for the null space and a basis for the row space of the matrix: $A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 2 & 6 \\ 5 & 5 & 15 \end{bmatrix}$

$A\vec{x} = \vec{0}$: $\left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 2 & 2 & 6 & 0 \\ 5 & 5 & 15 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

If $\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\quad x = -y - 3z$, y = free, z = free

$$\boxed{\{[1 \ 1 \ 3]\} \text{ is a basis for RS(A)}}$$

Let $y = 0$, $z = 1$ $\qquad$ $y = 1$, $z = 0$

$$\boxed{\left\{ \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\} \text{ is a basis for NS(A)}}$$